clearly, the supremum of a set 'S' may or may not exist and in case it exists, It may or may not belong to S.

Def: The fact that supremum M is the smallest of all the upper bounds of S may be described by the following two properties

$M = \sup S$
(i) M is the upper bound of S
(ii) for any $\epsilon > 0$ $\exists$ $y \in S$ s.t. $M - \epsilon < y$.

(i) M is the upper bound of S; i.e, $x \le M, \forall x \in S$.

(ii) No number less than M can be an upper bound of S, i.e for any +ve number ϵ, however small, $\exists$ a number $y \in S$ such that

$$y > M - \epsilon.$$

→ Again, It may be seen that a set cannot have more than one supremum.

for, let if possible M and M' be two suprema of a set S, so that M and M' are both upper bounds of S.

Also, M is l.u.b (supremum) and M' is an upper bound of S

$\therefore$ $M \le M'$ —— ①

Again, M' is the l.u.b (supremum) and M is an upper bound of S.

$\therefore$ $M' \le M$ —— ②

from ① & ②, it follows that $M = M'$

→ If the set of all lower bounds of a set S has the greatest member, say m, then m is called the greatest lower bound (g.l.b) or ~~of~~ the infimum of S.

Like the supremum, the infimum of